1 Additional Exercises

1.1 Counting Formula

1. Compute the order of the group of symmetries of a dodecahedron, when orientation reversing symmetries such as reflections in planes, as well as rotations are allowed. Do the same for the symmetries of a cube.

2. Let $G$ be the group of rotational symmetries of a cube. Let $S_e, S_f, S_v$ be the sets of edges, faces and vertices of the cube, respectively. Let $H_e, H_f, H_v$ be the stabilizers of a particular edge, face and vertex, respectively. Determine the formulas that represent the decomposition of each of the three sets $S_e, S_f, S_v$ into orbits for each of the three subgroups.

1.2 Operations of a group on itself

1. Given a group $G$, does the mapping $f : G \times G \rightarrow G$ given by $f(g, x) = xg^{-1}$ define a group action of $G$ onto itself?

2. Determine the class equation for each of the following groups.
   
   (a) The quaternion group.
   (b) The Klein four group.
   (c) The dihedral group $D_5$.
   (d) The dihedral group $D_6$
   (e) The dihedral group $D_n$

1.3 Class Equation of Icosahedral Group

1. Identify the intersection $I \cap O$ when the dodecahedron and cube are as in Figure 2.7 which was passed out in class. Here, $I$ is the group of 60 rotational symmetries of the dodecahedron and $O$ is the group of 24 rotational symmetries of the cube.

2. Two tetrahedra can be inscribed into a cube $C$, each one using half of the vertices. Relate this to the inclusion $A_4 \subset S_4$. Here $S_4$ is the symmetric group of all permutations of the elements $\{1, 2, 3, 4\}$ and $A_4$ is the normal subgroup of $S_4$ consisting of the even permutations in $S_4$. Recall that a permutation $p$ is even if its matrix $P$ has the property that $\det(P) = 1$.

3. Prove or disprove: An abelian group is simple if and only if it has prime order.