Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. Only write on one side of each page.

"Personally, I’m always ready to learn, although I do not always like being taught.” – Winston Churchill

Problems

1. Let $G$ be the group of rotational symmetries of a cube $C$. Two regular tetrahedra $\Delta$ and $\Delta'$ can be inscribed in $C$, each using half of the vertices. What is the order of the stabilizer of $\Delta$?

2. Do one of the following.
   
   (a) Prove the formula $|G| = |Z(G)| + \sum |C|$ where the sum is over the conjugacy classes containing more than one element and $Z(G)$ is the center of $G$.
   
   (b) Rule out as many of the following as possible as Class Equations for a group of order 10.
   
      i. $1 + 1 + 1 + 2 + 5$
      ii. $1 + 2 + 2 + 5$
      iii. $1 + 2 + 3 + 4$
      iv. $1 + 1 + 2 + 2 + 2 + 2$

3. Let $Z(G)$ be the center of a group $G$. Prove that if $G/Z$ is a cyclic group, then $G$ is abelian and hence $G = Z(G)$.