Problems

1. Prove the icosahedral group has no subgroup of order 30.

2. Prove no group of order $p^l$, where $p$ is prime and $l > 1$, is simple.

3. List all subgroups of the dihedral group $D_4$, and divide them into conjugacy classes.

4. Do both of the following.
   
   (a) Let $H$ be a normal subgroup of a group $G$ of order 2. Prove $H$ is in the center of $G$.
   
   (b) Let $H$ be a normal subgroup of prime order $p$ in a finite group $G$. Suppose $p$ is the smallest prime dividing $|G|$. Prove $H$ is in the center of $G$.

5. Prove no group of order $p^2q$, where $p$ and $q$ are distinct primes, is simple.

6. Do one of the following.
   
   (a) Prove the only simple groups of order less than 60 are groups of prime order.
   
   (b) Classify all groups of order 33.
   
   (c) Classify all groups of order 18.