The Problems

1. Use the definition (the limit form) of derivative to find $f'(x)$ if $f(x) = \frac{1}{2x+1}$.

2. Below is the graph of a function on a grid. Assuming the grid lines are spaces 1 unit apart both vertically and horizontally, sketch the graph of the derivative function over the same interval. Use the same grid for your sketch.

3. State the definition of:
   (a) A function $f$ being continuous at $x = c$.
   (b) A function $f$ being differentiable at $x = c$.

4. Given the function $f(x) = \begin{cases} 
  x^2 - 6, & x < 2 \\
  -2, & x = 2 \\
  Ax - 12, & x > 2 
\end{cases}$
   (a) Determine, with explanation, a value of $A$ that makes $f$ continuous at $x = 2$ or explain why no such number $A$ exists.

5. Do one of the following.
(a) When working with the exponential function \( f(x) = 3^x \), some people prefer to use the function \( g(x) = e^{kx} \) where \( k = \ln(3) \). Use logarithm and exponential rules to show these are really the same function.

(b) Determine the **exact** values of each of the following.
   i. \( \arcsin(1) \)
   ii. \( \arctan(1) \)
   iii. \( \cos\left(\arccos\left(\sqrt{2}/2\right)\right) \)
   iv. \( \arcsin(\sin(12\pi)) \) [Be careful.]
   v. \( \exp(3\ln(4)) \).

6. Do **one** of the following.

   (a) Without using a calculator, determine the following limits. Be sure to briefly justify your answer.
      i. \( \lim_{x \to 0} \frac{\sin^2(x)}{x} \)
      ii. \( \lim_{x \to 0} \frac{1}{1+3^x} \)
      iii. \( \lim_{x \to 2} \frac{x^2 + 5x - 14}{3x^2 - 3x - 6} \)

   (b) Without using a calculator, determine the following limits. Be sure to justify your answer.
      i. \( \lim_{x \to 1} \frac{10}{1 + 2^{1/(x-1)}} \)
      ii. \( \lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{x^2}\right) \) Hint: \( \frac{\infty}{\infty} \) is a “be careful” (indeterminate) form.