The Problems

1. (8 points each) Evaluate the following derivatives. Do not simplify.
   
   (a) \( y = (x^3 + 1) \sin(x) \)
   
   (b) \( y = \frac{x^2 + 1}{1 + \sec(x)} \).
   
   (c) \( T = (2s^{-4} + 3s^{-2} + 2)^{-6} \).
   
   (d) \( f(x) = \sqrt{5x - 8} \).
   
   (e) \( g(x) = \ln(\sin(x^2 + 7)) \).
   
   (f) Evaluate \( \frac{d^4}{dx^4}[4x^3 - 2x^5] \).

2. (10 points) Use the quotient rule and the derivatives of \( \sin(x) \) and \( \cos(x) \) to show

   \[ \frac{d}{dx} \cot(x) = -\csc^2(x). \]

3. (8 points each) Use the following table of outputs for the functions \( f, f', g \) and \( g' \) to compute the indicated derivatives.

   \[
   \begin{array}{c|c|c|c|c|c}
   x & f & f' & g & g' \\
   \hline
   1 & -2 & -0.5 & 3 & 4 \\
   2 & -4 & -1 & 1 & -3 \\
   3 & 0 & 0 & 9 & 2 \\
   4 & 3 & 2 & 4 & 0 \\
   \end{array}
   \]

   (a) Find \( F'(4) \) if \( F(x) = f(x) - 3g(x) \)

   (b) Find \( H'(2) \) if \( H(x) = 2 + f(g(x)) \).

4. (15 points) Do one of the following.
(a) Suppose a pebble is thrown vertically upward from the top of a 800 foot high building with an initial velocity of 32 feet per second.

i. Find the height of the pebble at \( t = 3 \) s.
ii. Find the velocity of the pebble at \( t = 3 \) s.
iii. Find the velocity of the pebble when it hits the ground.
iv. Find the maximum height of the pebble.

(b) An object moves along a coordinate line with position at time \( t \) (seconds) given by \( x(t) = t + 2 \cos(t) \) (meters). Find those times \( t \) from 0 to \( \pi \) when the object is moving forward and also slowing down.

5. (11 points) Do one of the following.

(a) Each of the following limits represents the derivative of some function \( f \) at some number \( c \). State \( f \) and \( c \) in each case.

i. \[ \lim_{h \to 0} \frac{\sqrt{1 + h} - 1}{h} \]

ii. \[ \lim_{h \to 0} \frac{(2 + h)^3 - 8}{h} \]

iii. \[ \lim_{x \to 1} \frac{x^9 - 1}{x - 1} \]

(b) Prove for a differentiable function \( f \) and a constant \( c \),

\[ \frac{d}{dx} [cf(x)] = c \frac{d}{dx} [f(x)] \]

by using the (limit) definition of derivative and the fact

\[ f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h} \]