The Problems

1. (7 points each) Evaluate the following indefinite integrals.
   
   (a) \[ \int (2e^x + 4 \sec(x) \tan(x)) \, dx \]
   
   (b) \[ \int \frac{1}{t^2} \left( \frac{2}{t} - \frac{5}{t^3} \right) \, dt \]
   
   (c) \[ \int \frac{x^3 - \sqrt{x} + 1}{x} \, dx \]
   
   (d) \[ \int \frac{23}{\sqrt{1-x^2}} + e^x \, dx \]

2. (16, 3 points)

   (a) Find a formula \( a(k) \), \( k = 0, 1, 2, \cdots \) that gives the following sequence. Express your answer using the ‘bar’ \((k^n)\) notation.

   \[ 3, -3, 1, 33, 111, 253, 477, 801, 1243, 1821, 2553, \cdots \]

   Show all of your work.

   (b) Express your formula in the ‘nonbar’ \((k^n)\) notation but \textbf{DO NOT SIMPLIFY} your answer.

3. (20 points) Use the summation techniques from our discussion of sequences and/or Section 5.2 of our textbook to find the exact area bounded by the graph of \( y = x^2 + 2x \), the \( x \)-axis, and the vertical lines \( x = 0 \) and \( x = 3 \).

4. Do one of the following
(a) (20 points) Suppose that \( a(k), k = 0, 1, 2, \ldots \) is an arbitrary (but unknown) sequence and that \( A(k), k = 0, 1, 2, \ldots \) is one of the discrete antiderivatives of \( a(k) \) but we don’t know whether or not \( A(0) = 0 \). Show that

\[
A(n + 1) - A(0) = \sum_{k=0}^{n} a(k) = a(0) + a(1) + a(2) + \cdots + a(n).
\]

(b) (20 points) The following limit gives the exact area of a region in the plane. Carefully describe that region. *DO NOT EVALUATE* the limit.

\[
\lim_{n \to +\infty} \frac{n}{\sum_{k=1}^{n} \left[ 3 \left( 2 + \frac{4k}{n} \right)^3 + \left( 2 + \frac{4k}{n} \right)^2 + 5 \right]} = \frac{4}{n}.
\]

5. (20 points) An airplane has a constant acceleration while moving down the runway from rest. What is the acceleration of the plane at liftoff if the plane requires 900 feet of runway before lifting off at 88 ft/s?

**Useful Facts**

- \( \sum_{k=1}^{n} 1 = n \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2} \)

- \( \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \quad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4} \)

- \( k^n = k(k-1)(k-2)\cdots(k-n+1) \)

- \( D_k [k^n] = nk^{n-1} \) and if \( a(k) = k^n \), then \( A(k) = \frac{1}{n+1} k^{n+1} + C \)

- \( D_k [2^k] = 2^k \) and if \( a(k) = 2^k \), then \( A(k) = 2^k + C \)

- \( D_k [r^k] = (r - 1) r^k \) and if \( a(k) = r^k \) then \( A(k) = \frac{1}{r-1} r^k + C \)

\[
\begin{align*}
  k^0 & = 1 \\
  k^1 & = k \\
  k^2 & = k^2 + k^1 \\
  k^3 & = k^3 + 3k^2 + k^1 \\
  k^4 & = k^4 + 6k^3 + 7k^2 + k^1
\end{align*}
\]