The Problems

1. (20 points) Do one of the following.
   
   (a) The base of a solid is the region in the first quadrant bounded by the graphs of $y = x^2$ and $y = \sqrt{x}$. The cross sections of the solid perpendicular to the $x$-axis are semicircles whose diameters lie in the $xy$-plane. Find the volume of this solid.
   
   (b) Find the volume of the solid generated by revolving the region bounded by the curve $y = \frac{4}{x^2}$, the vertical line $x = 1$ and the horizontal line $y = \frac{1}{2}$ about the line $x = 2$.

2. (20 points) Do one of the following.
   
   (a) A rock climber is about to haul up 21 pounds of equipment that has been hanging beneath her on 90 feet of rope that weighs $0.3 \text{ lb ft}$. How much work will it take?
   
   (b) A storage tank is a right circular cylinder 20 feet long and 8 feet in diameter with a horizontal axis. If the tank is half full of olive oil weighing $57 \text{ lb ft}^3$, find the work done in emptying the tank through an outlet that is 6 feet above the top of the tank.

3. (20 points) Do one of the following.
   
   (a) Use integration by parts to prove the following reduction formula is true.
   $$\int \sec^n(x) \, dx = \frac{1}{n-1} \sec^{n-2}(x) \tan(x) + \frac{n-2}{n-1} \int \sec^{n-2}(x) \, dx.$$  

   (b) Use the Useful Information for Sequences provided below to find the formula for
   $$\sum_{k=0}^{n} k \cdot 5^k$$

4. (10 points each) Set up any four of the following to the point where the problem can be finished by citing a formula from the Integral Table Handout. Be sure to cite the appropriate formula(s). Do not use a calculator.
   
   (a) 
   $$\int \frac{1}{1 + \sqrt{y}} \, dy$$
(b) \[ \int x^3 \cos (x^2) \, dx \]

(c) \[ \int \frac{\sin (t)}{(3 + \cos (t))^2} \, dt \]

(d) \[ \int x \sqrt{x + 2} \, dx \]

(e) \[ \int \frac{x}{9 + 4x^4} \, dx \]

(f) \[ \int \frac{x + 3}{\sqrt{x^2 + 2x - 8}} \, dx \]

**Useful Information about Sequences**

<table>
<thead>
<tr>
<th>(D_k [k^n] = nk^{n-1})</th>
<th>(D_k [c^k] = (c - 1) c^k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>If (a(k) = k^n), then (A(k) = \frac{1}{n+1} k^{n+1})</td>
<td>If (a(k) = c^k), then (A(k) = \frac{1}{c-1} c^k)</td>
</tr>
<tr>
<td>If (D_k [A(k)] = a(k)), then (\sum_{k=0}^{n} a(k) = A(k)</td>
<td>_{0}^{n+1})</td>
</tr>
</tbody>
</table>

\[
\sum_{k=0}^{n} U(k) \ D_k [V(k)] = U(k) \ V(k) \bigg|_{0}^{n+1} - \sum_{k=0}^{n} V(k+1) \ D_k [U(k)] \quad \text{(Discrete Integration by Parts)}
\]