Directions: Cite any use of technology. For partial credit, if you are unsure of your answer to a problem be sure to describe what you do know and where you think your error is. Include a careful sketch of any graph obtained by technology in solving a problem. Only write on one side of each page.

The Problems

I ( 10 points each ) Do any three (3) of the following problems.

1. Write inequalities that describe the region consisting of all points between, but not on, the spheres of radius $r$ and $R$ centered at the origin, where $r < R$.

2. Find the area of the triangle formed by points $P(2,0,-3)$, $Q(3,1,0)$, $R(5,2,2)$.

3. If $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ are vectors in $\mathbb{R}^3$, state whether each expression is meaningful. If it is, state whether it is a scalar or a vector.
   (a) $\overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c})$
   (b) $\overrightarrow{a} \times (\overrightarrow{b} \cdot \overrightarrow{c})$
   (c) $\overrightarrow{a} \times (\overrightarrow{b} \times \overrightarrow{c})$
   (d) $(\overrightarrow{a} \cdot \overrightarrow{b}) \times \overrightarrow{c}$
   (e) $(\overrightarrow{a} \cdot \overrightarrow{b}) \times (\overrightarrow{c} \cdot \overrightarrow{d})$
   (f) $(\overrightarrow{a} \times \overrightarrow{b}) \cdot (\overrightarrow{c} \times \overrightarrow{d})$

4. Identify the following quadric surfaces by name (e.g., sphere, hyperboloid of one sheet, et cetera).
   Do Not Sketch.
   (a) $y^2 + z^2 = 1 - 4x^2$
   (b) $y^2 + z^2 = x$
   (c) $y^2 + z^2 = 1$
   (d) $y = z^2 - x^2$
   (e) $y^2 + z^2 = 1 + x^2$
   (f) $4x^2 - y^2 + z^2 + 8x + 8z = -20$

II ( 15 points each ) Do any two (2) of the following problems.

1. Suppose $\overrightarrow{a}$ is a three-dimensional unit vector in the first octant that starts at the origin and makes angles of $60^\circ$ and $72^\circ$ with the positive $x$- and $y$- axes, respectively. What are the components of $\overrightarrow{a}$?

2. Given non-zero vectors $\overrightarrow{a}$, $\overrightarrow{b}$ for which Proj$_{\overrightarrow{a}} \overrightarrow{b}$ is also non-zero, show that the vector $\overrightarrow{b} - \text{Proj}_{\overrightarrow{a}} \overrightarrow{b}$ is orthogonal to $\overrightarrow{a}$. Do not give a geometric argument: use the dot product.

3. Suppose $L$ is the line that passes through the point $P(0,2,-1)$ and is parallel to the line with parametric equations $x = 1 + 2t$, $y = 3t$, $z = 5 - 7t$. Find the points where $L$ meets the three coordinate planes.
4. Write an equation in parametric form for the line of intersection of the planes \( x + y + z = 1 \) and \( x + z = 0 \).

\textbf{III} (20 points each) Do any two (2) of the following problems.

1. Write an equation for either of the planes that are parallel to the plane \( x + 2y - 2z = 1 \) and are two units away from it.

2. If \( \overrightarrow{a} \cdot \overrightarrow{c} = \overrightarrow{b} \cdot \overrightarrow{c} \), must it be the case that \( \overrightarrow{a} = \overrightarrow{b} \)? If so, explain why. If not, provide a counterexample.

3. Find an equation for the plane that contains the parallel lines
\[
\frac{x - 3}{2} = \frac{y + 4}{5} = \frac{3 - z}{6} \quad \text{and} \quad \frac{x + 4}{2} = \frac{y - 7}{5} = \frac{z + 1}{6}.
\]

4. Find parametric equations for the line through the point \((0,1,2)\) that is parallel to the plane \( x + y + z = 2 \) and perpendicular to the line \( x = 1 + t, \ y = 1 - t, \ z = 2t \).