The Problems

1. Find all maxima, minima and saddle points of

\[ f(x, y) = 9x^3 + \frac{y^3}{3} - 4xy. \]

2. Find the absolute maximum and minimum of the function

\[ f(x, y) = x^2 - xy + y^2 + 1 \]

on the closed triangular plate in the first quadrant bounded by the lines \( x = 0, \ y = 4, \ y = x \).

3. Find the points on the ellipse \( x^2 + 2y^2 = 1 \) where \( f(x, y) = xy \) has its extreme values.

4. Sketch the region of integration and then evaluate

\[ \int_{\ln 8}^{\ln 8} \int_{\ln y}^{\ln y} e^{x+y} \, dx \, dy. \]

5. Find the volume of the region that lies under the paraboloid \( z = x^2 + y^2 \) and above the triangle enclosed by the lines \( y = x, \ x = 0, \) and \( x + y = 2 \) in the \( xy \)-plane.

6. Find the volume of the solid in the first octant bounded by the coordinate planes, the plane \( x = 3 \) and the parabolix cylinder \( z = 4 - y^2 \).

7. Find the moment of inertia about the \( y \)-axis of a thin plate (lamina) bounded by the line \( y = 1 \) and the parabola \( y = x^2 \) if the density is \( \rho(x, y) = y + 1 \).

8. Use double integrals to find the volume of a sphere of radius \( R \).

9. A flat circular plate has the shape of the region \( x^2 + y^2 \leq 1 \). The plate, including the boundary, is heated so the temperature at the point \( (x, y) \) is

\[ T(x, y) = x^2 + 2y^2 - x. \]

Find the temperatures at the hottest and coldest points on the plate.
10. Rewrite (but do not evaluate) the triple integral
\[ \int_{0}^{1} \int_{-1}^{0} \int_{y}^{2} \, dz \, dy \, dx \]
in the order
\( (a) \, dy \, dz \, dx \)
\( (b) \, dx \, dz \, dy \)

11. Find the average value of \( F(x, y, z) = x^2 + y^2 + z^2 \) over the cube in the first octant bounded by the coordinate planes and the planes \( x = 1, \, y = 1, \) and \( z = 1 \).

12. Use Lagrange multipliers to find the point on the surface \( z = xy + 1 \) that is nearest the origin.

13. If \( x \) thousand dollars is spent on labor and \( y \) thousand dollars is spent on equipment, the output of a certain factory may be modeled by
\[ Q(x, y) = 60x^{1/3}y^{2/3}. \]
units. Assume $120,000 is available.

How should money be allocated between labor and equipment to generate the largest possible output?

14. In the above problem, use the Lagrange multiplier \( \lambda \) and differentials to estimate the change in the maximum output of the factory that would result if the money available for labor and equipment is increased by $1,000.

15. Over what region in the \( xy \)-plane does
\[ \iint_{R} \left( 4 - x^2 - 2y^2 \right) \, dA \]
have its maximum value?

16. Find the area enclosed by one leaf of the rose \( r = 12 \cos (3\theta) \).

17. Find the volume of the region in the first octant bounded by the coordinate planes and the planes \( x + z = 1, \) and \( y + 2z = 2. \)