Mathematics 232-A
Exam 3
Spring 2006

March 30, 2006

Name

Technology used:

Directions:

• Only write on one side of each page.

• Use terminology correctly.

• Partial credit is awarded for correct approaches so justify your steps.

Do any three (3) of these computational problems

C.1. Option: Find the inverse of the following matrix by hand. You may not use a calculator.
\[
\begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
1 & 2 & 2 & 2 & 2 \\
1 & 1 & 3 & 3 & 3 \\
1 & 1 & 1 & 4 & 4 \\
1 & 1 & 1 & 1 & 5 \\
\end{bmatrix}
\]

C.2. Write \( A = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} \) as a product of elementary matrices.

C.3. Two matrices \( A \) and \( B \) commute if \( AB = BA \). Show that the set of matrices in \( M_{22} \) that commute with \( \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \) is a subspace of \( M_{22} \) and find a basis for that subspace.

C.4. Let \( S = \{ \vec{v}_1, \vec{v}_2, \ldots, \vec{v}_p \} \) be a collection of vectors in a vector space \( V \). Show that the span of \( S, < S > \) is a subspace of \( V \) and that \( \dim(V) \leq p \).

Do one (1) of these problems from the text, homework, or class.

You may NOT just cite a theorem or result in the text. You must prove these results.

M.1. Suppose \( S = \{ \vec{v}_1, \ldots, \vec{v}_t \} \) is a basis for a vector space \( V \) and \( \vec{w} \neq \vec{0} \) is a vector in the span of \( S, < S > \). Prove there is a basis, \( T, \) of \( V \) where \( \vec{w} \in T \).

M.2. Suppose \( A \) is an invertible matrix of size \( n \). Prove that \( (A^{-1}) = (A')^{-1} \).

Do one (1) of these problems you’ve not seen before.

T.1. Let \( V \) be a vector space and \( U \) and \( V \) subspaces of \( W \). Show that the set of vectors \( U + V = \{ \vec{u} + \vec{v} \in W : \vec{u} \in U \text{ and } \vec{v} \in V \} \) is a subspace of \( W \).

T.2. If \( A \) is an invertible matrix of size \( n \), prove \( \det(A^{-1}) = \frac{1}{\det(A)} \).
Do this mathematical induction problem

Induct Use mathematical induction to prove the following.

Let $A$ be a square matrix of size $n \geq 2$ and $B$ the matrix obtained after multiplying each entry of row $i$ of $A$ by the nonzero constant $\alpha$ (a type 2 elementary row operation). Use the technique of mathematical induction to prove that $\det(A) = \frac{1}{\alpha} \det(B)$. 