Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. Only write on one side of each page.

Problems

1. (20 points) Using any previous results, prove part (b) of Proposition 3.21. Given angles $\angle P, \angle Q, \angle R$. If $\angle P < \angle Q$ and $\angle Q \cong \angle R$, then $\angle P < \angle R$.

2. (20 points) Using any previous result, prove the following portion of Proposition 4.4. Every angle has a bisector (do NOT show the bisector is unique.)

3. (20 points each) Do any three (3) of the following.

   (a) In the following interpretation, all incidence axioms and the first three betweenness axioms hold. Explain why, Proposition 3.4 fails. Use the usual Euclidean model except for three points $A, B, P$ where $P$ is between $A$ and $B$ in the usual Euclidean sense. For these three points re-interpret between to mean “$A$ is between $P$ and $B$”.

   (b) Using any previous result, prove Proposition 3.20 (Angle Subtraction). Given $\overrightarrow{BG}$ between $\overrightarrow{BA}$ and $\overrightarrow{BC}$, $\overrightarrow{EH}$ between $\overrightarrow{ED}$ and $\overrightarrow{EF}$, $\angle CBG \cong \angle FEH$, and $\angle ABC \cong \angle DEF$. Then $\angle GBA \cong \angle HED$.

   (c) Using any result through Chapter 4 prove the following. Let $\gamma$ be a circle with center $O$, and let $A$ and $B$ be two points on $\gamma$. The segment $AB$ is called a chord of $\gamma$. Let $M$ be the midpoint of segment $AB$. Prove that if $O \neq M$, then the perpendicular bisector of segment $AB$ passes through the center $O$ of $\gamma$.

   (d) Using any result through the corollaries to Theorem 4.3, prove the following.

    If $A * B * C$ and $\overrightarrow{DC} \perp \overrightarrow{AC}$ then $AD > BD > CD$. (See the figure on the board.)