Linear Independence

In class we showed

- If \( \{w_1, w_2, w_3\} \) is a linearly independent set in \( \mathbb{C}^{23} \), then the set

\[
\{2w_1 + w_2 + 3w_3, -3w_1 + 2w_2 + 4w_3, w_1 - 3w_2 - 7w_3\}
\]

is linearly dependent.

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\{2w_1 + w_2 + 3w_3, -3w_1 + 2w_2 + 4w_3, w_1 + 2w_2 + 3w_3\}
\]

is linearly dependent.

Answer both of the following questions.

1. Suppose that \( \{w_1, w_2, w_3\} \) is a linearly independent set in \( \mathbb{C}^{23} \), Is the set

\[
\{2w_1 + w_2 + 3w_3, -3w_1 + 2w_2 + 4w_3, w_1 + 2w_2 + 3w_3\}
\]

linearly independent?

2. Suppose that \( \{w_1, w_2, w_3\} \) is a linearly dependent set in \( \mathbb{C}^{23} \), Is the set

\[
\{2w_1 + w_2 + 3w_3, -3w_1 + 2w_2 + 4w_3, w_1 - 3w_2 - 7w_3\}
\]

linearly independent?

You might find the following matrix information useful.

\[
\begin{bmatrix}
2 & -3 & 1 \\
1 & 2 & -3 \\
3 & 4 & -7 \\
\end{bmatrix}
\xrightarrow{RREF}
\begin{bmatrix}
1 & 0 & -1 \\
0 & 1 & -1 \\
0 & 0 & 0 \\
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
2 & -3 & 1 \\
1 & 2 & 2 \\
3 & 4 & 3 \\
\end{bmatrix}
\xrightarrow{RREF}
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]