Math 258 – Fourth Hour Exam – Spring, 2004

Name _______________________________

Show your work. Partial credit will be given where appropriate. 16 points per problem

Use the following function for problems 1-2.

\[ f(x, y) = 7x^2 - 5xy + y^2 + x - y + 6 \]

1. a. Find \( f(2,3) \)

b. Find \( \frac{\partial f}{\partial x} \)

c. Find \( \frac{\partial f}{\partial y} \)

d. Find \( \frac{\partial^2 f}{\partial x \partial y} \)

2. Find all points \((x,y)\) where \(f(x,y)\) has a possible relative maximum or minimum. Use the second-derivative test to determine the nature of \(f(x,y)\) at each of these points.
3. Public health officials in a northern state are concerned with the death rate in their state. Suppose that the officials have approximated the death rate during the winter months as a function $f(x, y, z)$ where $x$ is the average daily temperature, $y$ is the number of days of snow during the period and $z$ is the number of available emergency medical workers.

a) Explain why you would expect $\frac{\partial f}{\partial x}$ to be negative.

b) Explain why you would expect $\frac{\partial f}{\partial y}$ to be positive.

c) Would you expect $\frac{\partial f}{\partial z}$ to be positive or negative? Why?

4. Approximate the area bounded by the graph of the function $f(x) = x^3$ and the x-axis between $x = 3$ and $x = 4$. Use a Riemann sum with 4 subintervals and use the right endpoints of the subintervals to approximate this area. Draw a picture of the graph of $f(x)$. Shade the region whose area you computed in the Riemann sum.
5. Find:

a) \[ \int_{0}^{4} (x^3 + 2) \, dx \]

b) \[ \int \left[ \frac{\sqrt{t}}{4} - 4(t - 3)^2 \right] \, dt \]

c) \[ \int e^{-x} \, dx \]

6. Recall that the Cobb-Douglas production function is \( f(x, y) = Cx^A y^{(1-A)} \) where \( f(x, y) \) is units of production, \( x \) is units of labor, \( y \) is units of capital and \( C \) and \( A \) are constants. Suppose for a particular production line, the Cobb-Douglas production function is \( f(x, y) = 25(x)^{\frac{2}{3}} (y)^{\frac{1}{3}} \)

a) Show that, if there are no units of labor available, production will be 0.

b) Suppose labor costs $50 per unit and capital costs $75 per unit. Write the cost function \( C(x, y) \) that shows the cost of production when \( x \) units of labor and \( y \) units of capital are used.

c) Use the technique of Lagrange multipliers to find the maximum level of production on this line when $1350 are available for labor and capital.
Extra Credit: What’s wrong with the Mariners?

- Bad pitching
- Bad hitting
- There’s something wrong with the Mariners?
- Who are the Mariners?