1. (10 pts.)
   a. Give a formal definition of what it means that the function \( f \) is \( \mathcal{O}(g) \).
   
   b. Give an informal definition of what it means to say that a function \( f \) is \( \_\mathcal{O}(g) \).

2. (10 pts.) Find witnesses to demonstrate that \( 4x^2 + 5x \ln(x) + 127 \) is \( \mathcal{O}(x^2) \). Show your work. Remember that \( \ln(x) < x \) for all \( x > 1 \).
3. (5 pts.) Using limits, demonstrate that $x \ln(x)$ is $O(x^2)$.

4. (10 pts.) Using Euclid’s algorithm, find the GCD of 84 and 105. Show your work.
5. (10 pts.) Find the internal representation (two’s complement) of -125. Give your (32 bit) answer in hex.

6. (10 pts.) Calculate (showing work as appropriate)

\[
\begin{array}{c}
1101 \\
+110 \\
-----
\end{array}
\]

\[
\begin{array}{c}
1101 \\
-111 \\
-----
\end{array}
\]

\[
\begin{array}{c}
1101 \\
x110 \\
(product) \\
-----
\end{array}
\]
7. (5 pts.) Calculate the following arithmetic expressions mod 10. Express your answer using the smallest positive integer.

\[6 + 7\]

\[6 \times 7\]

8. (15 pts.) Suppose that 

\[
\begin{bmatrix}
2 & 0 & 1 \\
1 & 0 & 0 \\
3 & 0 & 0 \\
\end{bmatrix}
\quad \text{and that} \quad
\begin{bmatrix}
3 & 0 & 0 \\
2 & 1 & 0 \\
0 & 1 & 0 \\
\end{bmatrix}
\]

Calculate

\[A + B\]

\[A \times B \text{ (matrix product)}\]
9. (5 pts.) Briefly state the technique for proving a statement using mathematical induction

10. (15 pts.) Use mathematical induction to show that

\[ \sum_{k=1}^{n} k \cdot (k!) = (n + 1)! \]

Using the following steps:

a. State and prove the base case

(continued on the next page)
b. State the inductive step as it applies to this problem.

c. What is the inductive hypothesis (or assumption)?

d. Complete the proof by proving the inductive step.

11. (5 pts.) Say something (appropriate to the course) about one of the following:

   a. Marin Mersenne
   b. Karl Friedrich Gauss
   c. Euclid
   d. Pierre de Fermat