Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. Only write on one side of each page.

“The one real object of education is to have a man in the condition of continually asking questions.”
-Bishop Mandell Creighton

Problems

1. Do both of the following:
   
   (a) Prove that if $G$ is a group with the property that the square of every element is the identity, then $G$ is abelian.
   
   (b) Let $G$ be a finite group. Show that the number of elements $x$ of $G$ such that $x^3 = e$ is odd. Show that the number of elements $x$ of $G$ for which $x^2 \neq e$ is even.

2. Do any two of the following
   
   (a) Prove that every subgroup of a cyclic group is cyclic.
   
   (b) Prove that the set of elements of finite order in an abelian group is a subgroup.
   
   (c) If $H$ and $K$ are subgroups of a group $G$, show that $H \cap K$ is a subgroup of $G$. Adapt your proof to show that the intersection of any number of subgroups of $G$, finite or infinite, is again a subgroup of $G$. Notational hint: Let $C$ be a collection of subgroups of $G$. Then we can denote the intersection of all the subgroups in $C$ by

   $\bigcap_{H \in C} H$

3. Optional Problem: Show by example that the product of elements of finite order in a nonabelian group need not have finite order.