1 Mathematics 433  Fall 2000  Problems to Turn in: 3

September 7, 2000

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

"You don’t understand anything until you learn it more than one way.” – Marvin Minsky

Problems

1. You must do this problem.

   (a) If $H$ is a subgroup of $G$, then by the **centralizer** $C(H)$ of $H$ we mean the set \( \{ x \in G : xh = hx \text{ for all } h \in H \} \).

   Prove that $C(H)$ is a subgroup of $G$.

   (b) Must the centralizer of an element of a group be Abelian?

   (c) Must the center of a group be Abelian?

2. Do one (1) of the following.

   (a) Suppose that $G$ is a group of order 16 and that, by direct computation, you know that $G$ has at least nine elements $x$ such that $x^8 = e$.

      i. Can you conclude that $G$ is not cyclic?

      ii. What if $G$ has at least five elements $x$ such that $x^4 = e$?

      iii. Generalize your results as a reasonable conjecture.

   (b) If $G$ is an Abelian group and contains cyclic subgroups of orders 4 and 5, what other sizes of cyclic subgroups must $G$ contain?