The beginning of wisdom is the definition of terms. - Socrates

When taking the examination, be sure to give as complete answers as you can. You need not provide as much detail as that necessary in a homework assignment but your solutions should be complete and easy to follow.

I 5 points
Give an example of a conditional statement, its converse and its contrapositive.

II 5, 5 points
1. Which parallel property does this logical statement encode?
\[
\forall l \quad \forall P \quad \forall m \quad (\sim (P_l) \land (P_m) \land (m \neq l)) \implies (\exists Q \ni (Q_l) \land (Q_m))
\]
2. What is the negation of the above logical statement.

III 15 points
Using any result up to and including Proposition 2.4, prove the following.
For every point P there exist at least two lines through P.

IV 20 points
Using any previous result, prove the following portion of Pasch’s Theorem.
If A, B, C are distinct noncollinear points and l is any line intersecting AB in a point between A and B, then l also intersects either AC or BC.

V 20 points
Let \( \mathcal{M} \) be a finite projective plane. Prove that all lines in \( \mathcal{M} \) have the same number of points lying on them.

VI 20 points
Using any result up to and including Proposition 3.2, prove the following part of Proposition 3.3.
If A * B * C and A * C * D, then A, B, C, D are distinct, collinear points.

VII 10 points
Use the Forward-Backward method to analyze the following problem (the converse of the Same Side Lemma)
Given that A, B, C are collinear points, l is a line other than \( \overrightarrow{AC} \) passing through C and points A and B are on the same side of l. Then A * B * C.