1. **Math 434: Problem Set 4**

1.1 **Finite Abelian Groups**

1. Find all the abelian groups of order less than or equal to 40 up to isomorphism.

2. Let $G, H, K$ be finite abelian groups.
   
   (a) Prove if $G \times H \cong G \times K$ then $H \cong K$.
   
   (b) Give a counterexample to show the above cannot be true in general.

3. Do both of the following.
   
   (a) What is the smallest positive integer $n$ such that there are exactly four nonisomorphic abelian groups of order $n$?
   
   (b) Show there are two abelian groups of order 108 that have exactly four subgroups of order 3.

4. Characterize those integers $n$ such that the only abelian groups of order $n$ are cyclic.

5. Determine the isomorphism class of $Aut(\mathbb{Z}_2 \times \mathbb{Z}_3 \times \mathbb{Z}_5)$.