Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

“Civilization advances by extending the number of important operations which we can perform without thinking of them.” (Alfred North Whitehead)

1 Problems

1. Do one of the following.
   
   (a) Determine the structure of the ring $\mathbb{Z}[x]/(x^2 + 3, p)$ where
   
   i. $p = 3$
   ii. $p = 5$

   (b) Fully describe the ring $\mathbb{Z}[i]/(2 + i)$.

2. Fully describe the ring obtained from $\mathbb{Z}$ by adjoining an element $\alpha$ satisfying the two relations $2\alpha - 6 = 0$ and $\alpha - 10 = 0$.

3. Suppose we adjoin an element $\alpha$ to $\mathbb{R}$ satisfying the relation $\alpha^2 = 1$. Prove the resulting ring is isomorphic to the product ring $\mathbb{R} \times \mathbb{R}$, and find the element of $\mathbb{R} \times \mathbb{R}$ which corresponds to $\alpha$.

4. Let $\alpha$ denote the residue of $x$ in the ring $R' = \mathbb{Z}[x]/(x^4 + x^3 + x^2 + x + 1)$. Compute the expressions for $(\alpha^3 + \alpha^2 + \alpha)(\alpha + 1)$ and $\alpha^5$ in terms of the basis $(1, \alpha, \alpha^2, \alpha^3, \alpha^4)$.

5. Do one of the following.

   (a) In each case describe the ring obtained from $\mathbb{Z}$ by adjoining an element $\alpha$ satisfying the given relation.
   
   i. $\alpha^2 + \alpha + 1 = 0$
   ii. $\alpha^2 + 1 = 0$

   (b) Let $R = \mathbb{Z}/(10)$. Determine the structure of the ring $R'$ obtained from $\mathbb{Z}$ by adjoining element $\alpha$ satisfying each relation.
   
   i. $2\alpha - 6 = 0$
   ii. $2\alpha - 5 = 0$.

6. Describe the ring obtained from $\mathbb{Z}/12\mathbb{Z}$ by adjoining an inverse of 2. In particular, what ‘standard’ ring is isomorphic to $\mathbb{Z}/12\mathbb{Z}$?